Exploring and updating the mathematical model behind the olfactory sense

Radu Cimpean[1]

1.Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, The Netherlands

Summary. Communication is a key aspect when it comes to the existence and functioning of systems. For the biological systems, neurons fulfil this role via chemical and electrical modes. There is an exchange of chemical substances that either excites or inhibits the relevant components till a spike, also called action potential, is reached and thus the neuron has relayed the information. As a system of interest, the research will focus further on the olfactory system.

The basic function of the olfactory system is represented by the identification of odors as singular or mixed chemical compounds. Given the amount of research conducted on different species, the first question to be asked is the following: Is there common ground regarding the olfactory sense across different classes of animals? As an answer, several articles offer some details on the matter [1], [6], [4]. According to [1], after reviewing carefully the existing literature, came to the conclusion that "notwithstanding mechanistic differences between species, the general principles of olfactory organization are shared by many animals". This means that the organization of the olfactory pathway displays striking similarities and it spans across a broad phylogenetical array of animals (from insects to vertebrates and even humans). Similar conclusions have been reached by [6], [4]. and by taking into account these findings, we can infer certain commonalities when discussing the olfactory system across species. This allows us to make use of literature sources that used different species (e.g. fruit flies, zebrafish) to build a general view on the olfaction principles. For a better understanding, the case of human olfaction will be further exemplified.

The chemical molecules are entering the nasal cavity and reach the olfactory epithelium, where the binding to the Olfactory Receptor Neurons (ORN) occurs, triggering a two pathway response. The information is further relayed to the olfactory bulb (OB). The exchange occurs in ball shaped components called glomeruli, where the closely related ORN synapse with other neurons, called mitral cells (MC), one for each glomerulus. The MC have a major role in conveying the odor information, through a process called whitening. Since there can be a lot of redundancy in the information gathered in the glomeruli from the ORN, the OB performs the above mentioned process with the purpose of decorrelating the activity patterns, aspect achieved via the interaction between the MC, which are excitatory by nature and granule cells (GC)/interneurons (IN), which are inhibitory. As a result, a preprocessed, decorelated pattern of signal activity is obtained, which is further fed to the higher areas of the brain for analysis and identification.

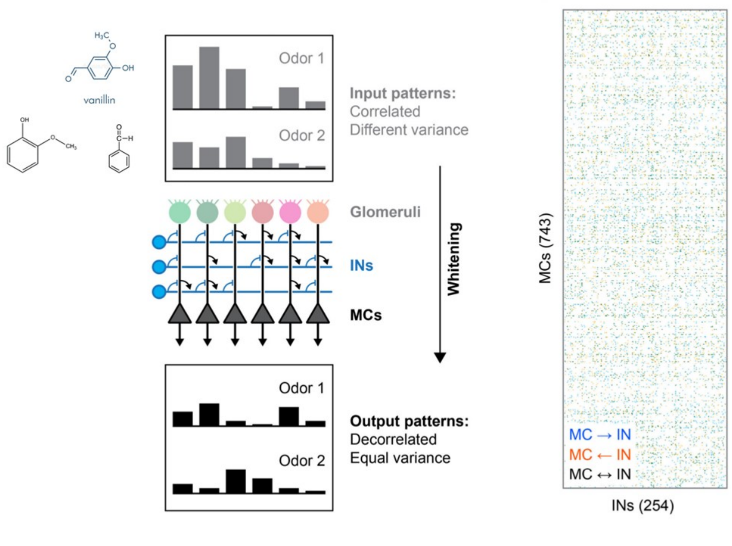

Figure 1: Whitening process in the olfactory bulb (left) and connectivity matrices(right) (adapted from [7].)

Given the above brief biological description of the olfactory process, the interest of this research lies in the mathematical modeling. The base model is provided by [5] in the form of equations of motion:

$$\dot{X} = -H_0 G_Y(Y) - \alpha_x X + I$$
$$\dot{Y} = W_0 G_X(X) - \alpha_Y Y + I_c,$$

where

- X-signal from the MC; Y-signal from the GC;
- G_X, G_Y cell output as continuous functions, proportional to the cell firing frequency;
- H_0 - $n \times m$ matrix describing the synaptic connection MC $\rightarrow$ GC; W_0 - $m \times n$ matrix describing the synaptic connection GC $\rightarrow$ MC;
- I - input, result of the sniffing process; I_c - input control, coming from higher areas of the brain;
- α_X, α_Y - parameters.

Moreover, [5] further indicates that the OB system may behave like a harmonic oscillator with damping, thus treating it as a group of coupled non-linear oscillators.

By reducing the biological complexity of the OB to the damped harmonic oscillator approach, several issues arise. One such issue is that the model does not account for the whitening process. This aspect can be confirmed by observing the diagrams in [7] and [2]. The outcomes indicated there do not fully match the proposed oscillator model. Another issue concerns the parameters (α_X, α_Y) for the OB system. For instance, in [5], they are taken as fixed values related to the firing rate of the neurons.

Given these issues, several research directions are being explored. As a starting point, it would be of interest to account for the whitening process by providing a function based approach to the parameters (α_X, α_Y). This would entail modeling them as characteristic functions, and further considering obtaining a well-posedness result, or simply put, getting a unique solution whose value will only modify slightly if we make a small change in the initial conditions. Another key aspect is the optimization of the connectivity matrices H_0 and W_0, again towards the goal of expressing the whitening process more accurately.

Therefore, it would be of interest to make changes to the above given model to incorporate the aforementioned biological findings that could be further used in analysing more accurately the behavior of the olfactory system. This will provide valuable insight into applications for an artificial nose, that could be used in detecting chemical indicators of certain diseases.

References

[1] B. Ache and J. Young, "Olfaction: Diverse species, conserved principles". Neuron, 48:417–430, 2005.

[2] M. Beyeler, F. Stefanini, H. Proske, G. Galizia and E. Chicca, "Exploring olfactory sensory networks: simulations and hardware emulation". Biomedical Circuits and Systems Conference (BioCAS), Paphos, Cyprus, 2010.

[3] L.Buck, "Unraveling the sense of smell (Nobel lecture)". Angewandte Chemie International Edition, 44:6128–6140, 2005.

[4] B. Kaupp, "Olfactory signalling in vertebrates and insects: differences and commonalities". Nature Review Neuroscience, 11:188–200, 2010.

[5] Z.Li and J.J.Hopfield, "Modeling the Olfactory Bulb and its Neural Oscillatory Processings". Biol. Cybern.,61:379-392, 1989.

[6] Y. Niimura, "On the origin and evolution of vertebrate olfactory receptor genes: Comparative genome analysis among 23 chordate species". Genome Biology and Evolution, 1:34–44, 2009.

[7] A. Wanner and R.W. Friedrich, "Whitening of odor representations by the wiring diagram of the olfactory bulb". Nat Neurosci.,23(3):433-442, 2020.