Networks of memristors and the effective memristor

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Summary. Memristors, originally introduced by L.O. Chua in 1971, are resistors with a memory storage. They are characterized as a relation between electrical charge and magnetic flux. In this work, we show that a network of memristors attached to an external source can be described by a single memristor. Moreover, we will see that this memristor can be characterized based on the dynamics of the individual memristors in the network and their orientation. This also enables the design of networks with desired port behavior.

1 Introduction

The emerging field of neuromorphic computing aims to reduce the energy requirements of computing platforms. It draws inspiration from the functioning of the brain and wants to learn from its energy efficiency [2]. For these new neuromorphic technologies dedicated hardware needs to be developed. It is suggested that memristors will play an important role within this new hardware. Memristors, originally introduced by Chua in [1], are resistors with a memory storage that can act as non-volatile memory. In addition, because of their dynamical nature, memristors have high potential of mimicking the behavior of certain elements of the brain.

In this work, we consider networks of memristors attached to an external source, and we show that such networks can be replaced by a so-called *effective* memristor. In addition, this effective memristor can be characterized based on the dynamics and orientation of individual memristors in the network. This result provides a stepping stone for the analysis of neuromorphic computing systems with memristors.

2 Main results

We will consider memristor networks that are attached to an external source through a port, see Figure 1 for an example. We denote the current through and the voltage across the port by $i_P(t)$ and $v_P(t)$, respectively. In addition, we define $q_P(t)$ and $\varphi_P(t)$ as the charge and flux at the port which satisfy

$$\tfrac{\mathrm{d}}{\mathrm{d}t} q_P(t) = i_P(t), \qquad \tfrac{\mathrm{d}}{\mathrm{d}t} \varphi_P(t) = v_P(t).$$

In this work, we want to show that the relation between $q_P(t)$ and $\varphi_P(t)$ can equivalently be described by a single effective memristor.

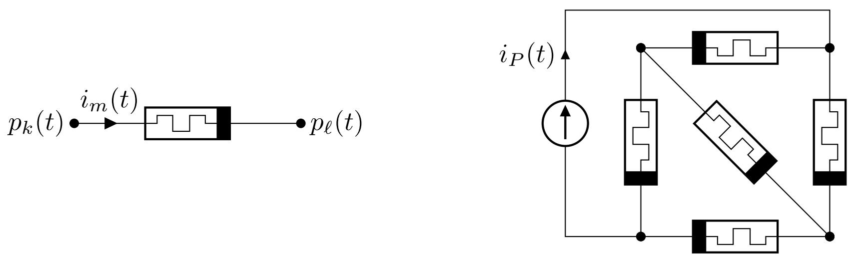

Figure 1: Left, a depiction of a memristor between a node k an ℓ. The voltage through this device is given by $v_m(t) = p_k(t) - p_\ell(t)$. Here, $p_k(t)$ and $p_\ell(t)$ denote the voltage potentials at node k and ℓ. Right, a network of memristors attached to a source applying an input $i_P(t)$ to the circuit.

Before we can do so, we provide the description of a single device. A memristor on a branch m in an electrical circuit can be depicted as in Figure 1. Here, we let $i_m(t) \in \mathbb{R}$ and $v_m(t) \in \mathbb{R}$ denote the current through and voltage across the device, respectively. The device can be characterized as a relationship between charge and flux, see [1]. Therefore, we define $q_m(t) \in \mathbb{R}$ and $\varphi_m(t) \in \mathbb{R}$ as the charge and flux at branch m, which satisfy

$$\tfrac{\mathrm{d}}{\mathrm{d}t} q_m(t) = i_m(t), \qquad \tfrac{\mathrm{d}}{\mathrm{d}t} \varphi_m(t) = v_m(t). \tag{1}$$

Then, a flux-controlled memristor is modeled as $\varphi_m(t) = h_m(q_m(t))$ for some (strongly) monotone function $h_m : \mathbb{R} \to \mathbb{R}$. We observe that, using (1), its behavior can be modeled as the dynamical system

$$v_m(t) = \frac{\mathrm{d}h_m(q_m(t))}{\mathrm{d}q_m} i_m(t), \qquad \tfrac{\mathrm{d}}{\mathrm{d}t} q_m(t) = i_m(t). \tag{2}$$

Here, $\frac{\mathrm{d}h_m(q_m)}{\mathrm{d}q_m}$ represents the memristance function of the device. It has the unit Ohm and its value varies depending on the history of the current i_m applied to the device. A typical example of a memristance function is depicted in Figure 2.

Now, we are able to derive the following result.

Theorem 1 *Consider a connected memristor network attached to an external source through a port. If all memristors in the network are described by* (2) *for some (strongly) monotone function* $h_m : \mathbb{R} \to \mathbb{R}$*, then there exists a (strongly) monotone functions* $h_M : \mathbb{R} \to \mathbb{R}$ *such that*

$$v_M(t) = \frac{\mathrm{d}h_M(q_M(t))}{\mathrm{d}q_M} i_M(t), \qquad \tfrac{\mathrm{d}}{\mathrm{d}t} q_M(t) = i_M(t).$$

Theorem 1 shows that there exists a relation between the charge and flux at the port of a network of memristors. We interpreted this results as that a network of memristors can be replaced by a so-called *effective* memristor. The behavior of this effective memristor is described by the function h_M. The observation that the port behavior of a network of memristors can be described as a single memristor was already made in [1, Theorem 2]. However, Theorem 1 provides an explicit description of this relation. This enables us to provide simulations of the behavior of the effective memristor of a network, for example in Figure 2 we show the memristance curve of the effective memristor of the network depicted in Figure 1.

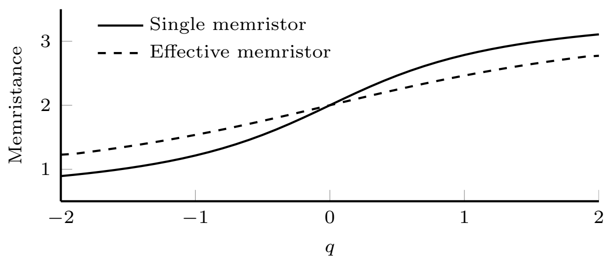

Figure 2: Depiction of a memristance function $\frac{\mathrm{d}h_m(q_m)}{\mathrm{d}q_m}$ for a single device, and that of the corresponding effective memristor of the network in Figure 1.

3 Discussion

The above result shows that the memristive behavior of a network of memristors can be described by a single effective memristor. This result is of interest when investigating materials that host memristors. These materials often include large networks of memristors instead of single devices. The small scale of these materials makes it complicated to measure the behavior of a single memristor while it is possible to measure the behavior of the network between certain points, e.g. by connecting these points to an external source. In addition to the analyses of the port behavior of networks of memristors our result can be used in the synthesis of networks of memristors in order to achieve desired port behavior.

These results can be a starting point for the analysis of network including both memristors and capacitors, which are of interest since materials often not only include memristors but also other elements such as capacitors.

References

[1] L. Chua. Memristor-the missing circuit element. *IEEE Transactions on Circuit Theory*, 18(5):507–519, 1971.

[2] C. D. James, J. B. Aimone, N. E. Miner, C. M. Vineyard, F. H. Rothganger, K. D. Carlson, S. A. Mulder, T. J. Draelos, A. Faust, and M. J. Marinella. A historical survey of algorithms and hardware architectures for neural-inspired and neuromorphic computing applications. *Biologically Inspired Cognitive Architectures*, 19:49–64, 2017.