Influence of annealing schemes on the success rate of Ising machines

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Summary. An Ising machine is a radical new computing platform that has the potential to solve hard optimization problems more efficiently than traditional digital computers. Unfortunately, for specific tasks, the relatively low success rate of finding the optimal solution remains an issue. Our research investigates novel annealing schemes that could drastically improve the success rate of these highly anticipated Ising machines.

Combinatorial optimization problems play an important role in different industries of our society. They can be found in logistics, drug development, electronic circuit-design (VLSI) and even in finance. However, traditional computing platforms require large amounts of time and resources to find the solution to such optimization problems, because the amount of possible solutions increases very strongly with the number of variables. To solve these optimization problems more efficiently, a renewed interest in novel computing platforms has emerged [1]. Our research group focuses on one of the most promising platforms, called the Ising machine (IM) [2].

An IM is a natural computing system that uses a network of analog artificial spins to emulate the Ising model of which the energy is given by $H_{Ising} = -\frac{1}{2}\sum_{ij} J_{ij}\sigma(x_i)\sigma(x_j)$, where $\sigma(x_i)$ represents the sign of the i-th spin amplitude x_i and J_{ij} the matrix that describes how the spins are interconnected. An optimization problem can then be encoded through this interconnection matrix and driven by their coupling, the spin network will tend to evolve towards its lowest energy state. This energy state (i.e. the ground state) then corresponds to the optimal solution of the encoded problem [1]. The artificial spin-network of an IM can be implemented in numerous ways, but all of them consist of three essential buildings blocks: a non-linearity, a spin coupling mechanism and a feedback mechanism. This feedback signal contains two contributions: the self-feedback and the mutual coupling feedback. To regulate their respective strengths, we use two parameters, which we call here *a* and *b*. Our group has proposed and build an opto-electronic IM (OE-IM) that is made out of opto-electronic oscillators (OEO) [2]. The optical part of the OEO includes a Mach-Zehnder modulator (MZM) that takes up the role of non-linear system. In electronic part, an FPGA is used for calculating the spin coupling signal which is then electrically fed back to the MZM. The evolution of a spin amplitudes x_i is described by the following equation:

$$\dot{x}_i = \cos^2(\,ax_i + b\sum_j J_{ij}\,x_j + \gamma - \,\pi/4) - 1/2$$

where $\pi/4$ and 1/2 are two bias terms, *a* and *b* are the previously discussed feedback strengths, J_{ij} represents again the interconnection matrix and γ the gaussian white noise.

Although IMs are promising, they still face a number of challenges. One difficulty is that the success rate of finding the optimal solution, which depends on the value of both *a* and *b*, can be relatively small [3]. Physically this means that the IM has a high probability of getting stuck in a local minimum, of which the spin configuration does not corresponds to the desired optimal solution. As a consequence, numerous runs are required in order to have at least one instance that reaches the ground-state. This strongly increases the average time-to-solution.

To tackle this problem, annealing schemes are being investigated as a tool to navigate around these local minima and increase the overall success rate of IMs [4]. These annealing schemes gradually change at least one feedback parameter of the system towards a certain target value. Annealing schemes typically operate by scanning the feedback strengths through the operating threshold of the IM, which depends on the value of both *a* and *b*, as there are less excited states near this region in which the IM could get stuck. However, there are various methods to do this. In this contribution, we use numerical simulations to study two of these schemes, namely linear annealing (LA) and linear adiabatic annealing (LAA).

In a LA scheme, at least one of the two feedback parameters increases linearly with time from a start value towards a target value. Although we keep the noise strength fixed in our simulation, its importance in the dynamics of the system will decrease if we drive the IM further away from the operating threshold by increasing a and/or b. Our simulations indicates that when such annealing is performed slow enough, it is possible to significantly increase the maximum success rate of some specific benchmark problems. However, as mentioned, annealing will increase the time that is needed to perform a single run, meaning there is a trade off between finding the ground state over several runs without annealing, or doing one long run with annealing. At the conference, we will discuss what the optimal ratio is between the annealing time and the success rate.

In the case of LAA, we change the Hamiltonian of a system over time instead of the value of the feedback parameters. At the start, the system is initialized in the (known) ground state of an easy-to-solve problem. Next, the system's Hamiltonian is transformed linearly over time, by changing the interconnection matrix elements J_{ij}, towards the Hamiltonian of the actual problem to be solved. This evolution is schematically illustrated in Fig.1. The idea behind this technique is that when the system is initialized in the ground state of the easy Hamiltonian, the system will remain in the ground state of an interpolated Hamiltonian if the transition happens slow enough. Eventually, the system should end up in the ground state of the target Hamiltonian. The principle of adiabatic evolution has already been proven in quantum mechanics but not in classical mechanics, which is the regime in which our IM operates. Therefore, we investigate in this contribution the computational performance of this scheme using the IM described above.

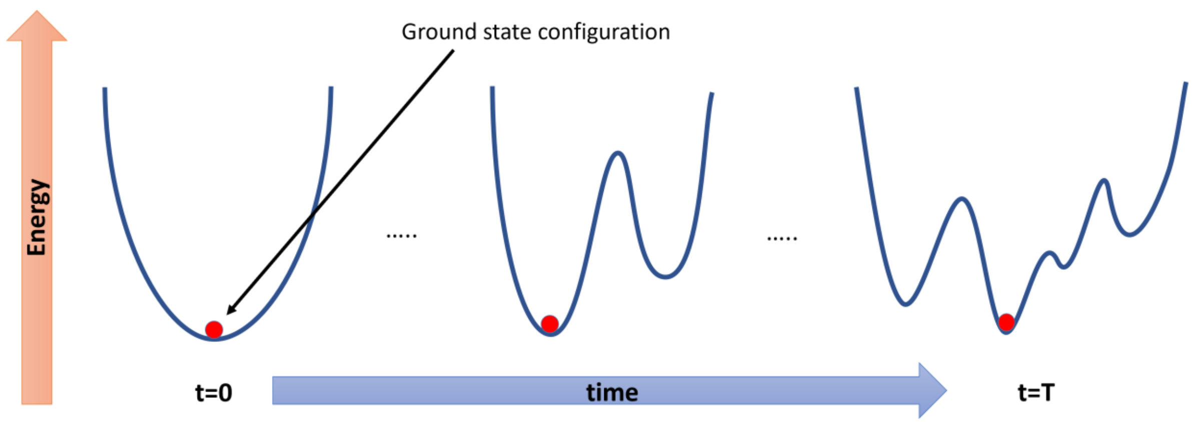

Figure 1. This figure shows a simplistic illustration of the concept of LAA. The system starts at the ground state configuration at t = 0 of a simple problem of which the ground state is known. As time increases, the energy landscape changes towards a complex landscape that contains more local minima.

We investigate if, and up to which extent, these types of annealing can be beneficial in classical IM by applying it to several benchmark tasks (e.g., biqmac-library, g-set). We quantify the performance based on the success rate of finding the optimal solution and the average time-to-solution. The code behind these numerical simulations is based on the dynamical equation of our OE-IM that was discussed above.

To summarize, our research uses numerical simulations to investigates how LA and LLA schemes near the operation threshold of an OE-IM could improve the success rate and time-to-solution of different benchmark problems.

References

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