Exploring quantum mechanical advantage for reservoir computing

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Summary. Quantum reservoir computing is an emerging field in machine learning with quantum systems. Here, we establish a link between quantum properties of a quantum reservoir, namely entanglement and its occupied phase space dimension, and its classical linear short-term memory capacity. We find that a high degree of entanglement in the reservoir is a prerequisite for a more complex reservoir dynamics that is key to unlocking the exponential phase space and higher short-term memory capacity. We quantify these relations and discuss the effect of dephasing on the performance of physical quantum reservoirs.

The machine learning paradigm of reservoir computing has proven to be well-suited for various applications such as autonomous learning and time-series prediction [1, 2]. While classically implemented reservoir computers [3] have been researched extensively, their quantum mechanical counterparts [4] only now are coming into the focus of the research community. The promises they bring to the table are, among others:

- exponential phase space dimension scaling with system size,
- entanglement as a resource with no classical analogue.

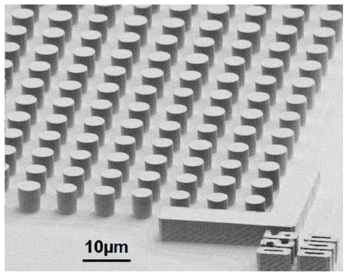

Figure 1: A possible physical implementation of a quantum reservoir computer: coupled-cavity arrays. Image taken from [5].

The study of quantum reservoir computers (QRC) is now more important than ever, as with the advent of sophisticated semiconductor fabrication techniques for quantum photonic systems like coupled-cavity arrays (see Fig. 1) QRCs are at the brink of realization. In preparation to this, we investigate the fundamental properties of QRCs on the basis of the transverse-field Ising model (TFIM). Using the linear short-term memory (STM) capacity C_{STM} as an established, classical benchmark, we measure the QRC's performance in relation to several of its key figures, specifically the TFIM coupling strength J_0, the mean logarithmic negativity $\bar{E}_{\mathrm{N}}$, and the covariance dimension D_{c} – the latter of which allow to probe the intrinsic quantum properties of the QRC [6].

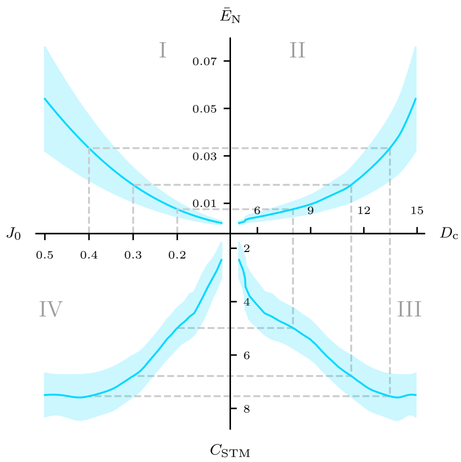

Figure 2: All relations between coupling strength J_0, mean logarithmic negativity $\bar{E}_{\mathrm{N}}$, covariance dimension D_{c} and linear short-term memory capacity C_{STM}.

The logarithmic negativity $E_{\mathrm{N}} \in [0, 1]$ is given by

$$E_{\mathrm{N}}(\rho) = \log_2 \left\| \rho^{\Gamma_{\mathrm{A}}} \right\|_1 , \tag{1}$$

where $\rho^{\Gamma_{\mathrm{A}}}$ is the partial transpose of ρ with respect to subsystem A and $\|\cdot\|_1$ denotes the trace norm. As an entanglement monotone, it provides a method for quantifying entanglement between any two bipartitions of the quantum system. Here, we calculate the average logarithmic negativity over all bipartitions and the whole reservoir dynamics to obtain a single number $\bar{E}_{\mathrm{N}}$ as a measure for the entanglement strength during execution of the short-term memory task.

The time evolution of the QRC state can be understood as a path in its 4^N-dimensional phase space, where N is the number of qubits in the system. To investigate, whether the QRC actually takes advantage of the full phase space or only evolves on a lower-dimensional manifold, we employ a measure called the *covariance dimension*. For this, random clusters from the QRC's discretely sampled state evolution are chosen and their spatial extent is approximated by a principal component analysis (PCA). By counting the number of principal components which are above a certain threshold, the covariance dimension for each cluster is then determined. The overall covariance dimension D_{c} is obtained by averaging over all individual covariance dimensions of the clusters.

As can be seen for the example of a 3-qubit TFIM in Fig. 2, all four aforementioned properties are directly linked to each other. Upon increasing the qubit coupling strength, we find a stronger mean entanglement of the QRC (see quadrant I) and a higher dimensionality of the phase space submanifold (see quadrant II). We explain this by the fact that the input injection method applied here for the execution of the STM task is given by projective measurement, which by definition destroys the entanglement of the input qubit with the rest of the system. As we keep the input injection frequency constant, the increased speed of the reservoir dynamics resulting from larger coupling strengths allows the quantum system to build up more entanglement and explore a higher dimensional phase space before the input injection collapses the state again. Interestingly, the stronger expression of these quantum properties also results in a better performance of the QRC in the classical linear short-term memory task as shown in quadrants III and IV of Fig. 2, respectively.

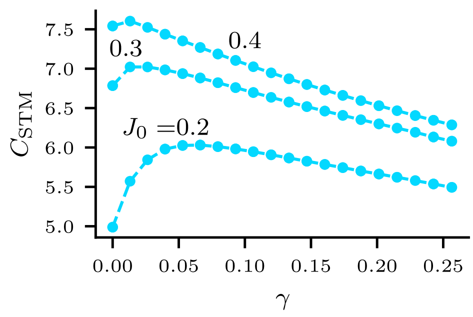

Figure 3: Influence of dephasing on the 3-qubit QRC for different coupling strengths.

In real-world scenarios, the interaction of the QRC with the environment plays an important role and should therefore be investigated systematically. For this matter, we introduce a channel to the system, which simulates a loss of quantum entanglement due to interaction with the environment. This so-called *dephasing channel* can be tuned by the *dephasing strength* γ. While we see a decrease in performance of strongly coupled TFIM-QRCs when increasing the dephasing, the opposite effect appears in the weakly coupled regime for a small amount of dephasing, as can be seen in Fig. 3. This poses the possibility of using the ubiquitous decoherence effects in noisy intermediate-scale quantum devices as a resource for quantum reservoir computing, which are otherwise undesired e.g. for gate based quantum computing.

In summary, we make a first step to assess the influence of quantum effects on the performance of QRCs in classical tasks. Future research will concern the question of how to translate between descriptions of coherent quantum systems on the one hand and nonlinear network dynamics on the other. We aim to identify effects of tunable parameters also for photonic semiconductor systems such as the coupled-cavity array.

References

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